Strategy Choices
Across the Life Span

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One of the most striking characteristics of human cognition is its variability. Both children and adults often possess multiple strategies, rules, concepts, and theories that they use to think about a given phenomenon or solve a given type of problem. For example, in such diverse domains as arithmetic, spelling, serial recall, and moral reasoning, children know and use multiple strategies. Recent trial-by-trial analyses have shown that the variability is present even in domains that have given rise to classic stage theories. Thus, when 5-year-olds are presented number conservation problems, they not only judge on the basis of the relative lengths of the rows, as stated in Piaget's theory and virtually all developmental psychology textbooks, but also sometimes rely on the type of transformation and other times rely on the results of counting (Siegler, 1995).

This variability is not just a cognitive curio, something that is true but without further ramifications. Rather, it appears to influence both performance and learning. With regard to performance, the greater the number of relevant rules, strategies, or conceptualizations that an individual can apply to a task, the more finely the person can fit the one they use on a particular occasion to task and situational demands. For example, in a study of preschoolers' arithmetic, children who already occasionally used the min strategy (counting from the larger addend) were able to solve more challenging problems than they had previously encountered (e.g., 2 + 21), whereas children who had not yet discovered the strategy were unable to adapt to the challenges posed by such problems and simply said
they couldn't solve them (Siegel & Jenkins, 1989). Similarly, possessing multiple ways of thinking about a task correlates positively with ability to learn more about it (Graham & Perry, 1993; Siegel, 1995).

Although cognitive variability can potentially enhance performance and learning, the degree to which it does so depends on how well children choose among the alternative strategies. If children know a faster strategy and a slower, more accurate one, they will benefit only if they choose the faster strategy when speed is most important and choose the more accurate one when accuracy is. Selecting randomly will yield worse performance than always using the approach that on average yields the better outcome. Thus, the benefits of cognitive variability depend on the quality of choices among alternatives.

How adaptive are these choices? Those interested in children's strategy choices have tended to reach quite negative conclusions. They have focused on findings that children who have been taught new strategies often do not use them later when they are free to choose (Ghatala, Levin, Pressley, & Goodwin, 1986; Keeney, Cannizzo, & Flavell, 1967), that children frequently choose alternatives with lower expected values over ones with higher values (Klayman, 1985), and that children who have been encouraged to plan commonly fail to do so (Schonlick & Friedman, 1987). This has led to conclusions such as the following:

Certainly one of the lessons from research across many areas of cognition is that children often do not use their available knowledge... A child might know that rehearsal is a good strategy for remembering a list of chores that need to be done, but not rehearse because he or she lacks time, energy, capacity, or motivation to exert the effort. (Flavell, Miller, & Miller, 1993, p. 261)

Despite these pessimistic impressions, an increasing body of evidence indicates that children's strategy choices are usually highly adaptive. The choices respond in reasonable ways both to inherent problem characteristics and to fluctuating situational demands. For example, children most often adopt particular strategies on problems in which those strategies are most advantageous. They also choose strategies in ways that respond to concerns of both speed and accuracy. Moreover, they respond appropriately to situational incentives to use one strategy rather than another. Table 4.1 lists some of the types of choices on which children have been found to choose adaptively and the domains in which they have been found to make them.

Research on the adaptiveness of children's strategy choices has focused almost entirely on children between 4 and 12 years of age. Related work on the adaptiveness of adults' strategy choices has focused almost entirely on the adaptiveness of the choices of college students and other youthful adults. However, Siegler (1995), among others, has claimed that the adaptiveness of strategy choices reflects basic features of human cognition, and that adaptive choices should therefore be evident in all age groups from infancy to old age. The goal of the present chapter is to test this hypothesis through examining strategy choices not only among children and young adults but also among infants and older adults.

The chapter is divided into four main parts. The first describes five basic phenomena regarding strategy choice, and illustrates them with examples from research on 4- to 12-year-old children. The second part describes a computer simulation, ASCM, that embodies a set of hypotheses about the mechanisms that underlie these five phenomena. Relying entirely on implicit knowledge, ASCM generates highly adaptive choices, similar to those that children make. The third part of the chapter focuses on the strategy choices of infants who need to go down ramps of varying steepness; many of the same basic phenomena emerge as with older children and the simulation model. The fourth part focuses on the strategy choices of senior citizens asked to solve mental multiplication problems; again, many of the same phenomena emerge. The overall objective is to show that throughout the lifespan, people choose adaptively among alternative strategies.
BASIC PHENOMENA RELEVANT TO STRATEGY CHOICES

Variability

Innumerable studies have depicted cognitive development in terms of a 1:1 correspondence between children's age and their way of thinking. Young children are said to think in one way, somewhat older children in a somewhat more sophisticated way, yet older children in a yet more sophisticated way. Contrary to this standard depiction, however, recent trial-by-trial analyses indicate that children of a single age often think about a given problem, concept, or phenomenon in a variety of ways.

One task in which such cognitive variability has been observed is elementary school children's strategies for solving single-digit addition problems. Examination of both videotaped records of ongoing behavior and immediately retrospective self-reports reveals five relatively common strategies (each used on between 1% and 36% of trials among the kindergartners, first graders, and second graders in Siegler, 1987a). Sometimes children use the sum strategy, in which they count from one; to solve 3 + 6, a child using the sum strategy might put up three fingers, then six more, then count from one to nine. Other times children use the min strategy, which involves counting from the larger addend the number of times indicated by the smaller addend. Here, they would solve 3 + 6 by counting "6, 7, 8, 9" or "7, 8, 9." On other occasions, they use decomposition, which involves translating the problem into an easier form and then making the necessary adjustment. A child solving 3 + 6 via decomposition might think, "3 + 7 = 10, 6 is 1 less than 7, so 3 + 6 = 9." Still other times, they use retrieval or guessing to generate an answer.

These diverse strategies are not artifacts of one child consistently using one strategy and another child consistently using a different one. The majority of kindergartners, first graders, and second graders use at least three of the five strategies; a substantial minority use more (Siegler, 1987a). This multiple strategy use is apparent within classes of similar problems, and even on the same problem presented to the same child on consecutive days. In two studies, one on addition (Siegler & Shrager, 1984) and one on telling (Siegler & McGilly, 1989), fully one-third of children used different strategies on the identical problem on two successive days. Only a small part of this day-to-day variability could be explained by learning, because the progression of strategies was not consistently from less to more advanced. For example, in the study of addition, almost as many children retrieved the answer on the first day and used the sum strategy on the second as did the reverse.

This strategy diversity is not limited to any particular domain. Consider just findings from our own studies. To multiply, 8- to 10-year-olds sometimes repeatedly add one of the multiplicands, sometimes write the problem and then recognize the answer, sometimes write and then count groups of hatch marks that represent the problem, and sometimes retrieve the answer from memory (Lemaire & Siegler, 1995). To tell time, 7- to 9-year-olds sometimes count forward from the hour by ones, fives, or both, sometimes count backwards from the hour by ones and/or fives, sometimes count from reference points such as the half hour, and sometimes retrieve the time that corresponds to the clock hands' configuration (Siegler & McGilly, 1989). To spell words, 7- and 8-year-olds sometimes sound out words, sometimes look them up in dictionaries, sometimes write out alternative forms and try to recognize which is correct, and sometimes recall the spelling from memory (Siegler, 1986). To serially recall lists of unrelated stimuli, 5- to 8-year-olds sometimes repeatedly recite the names of items within the list during the delay period, sometimes recite the names once and stop, and sometimes just wait (McGilly & Siegler, 1989, 1990).

Similar strategy diversity has been observed by other investigators in children's arithmetical (Cooney, Swanson, & Ladd, 1988), causal reasoning (Shultz, Fisher, Pratt, & Rulf, 1986), scientific reasoning (Schaufler, 1990), spatial reasoning (Olsson, 1984), referential communication (Kahan & Richards, 1986), language use (Kuczaj, 1977), and motor activity (Goldfield, 1994). These diverse strategies have been observed among adults as well as children, and in Japan and China as well as in North America and Europe (Geary, Fan, & Bow-Thomas, 1999; Kuhara-Kojima & Hatano, 1989). It has been observed in the order with which strategies are considered as well as in the strategies that are used (Reder, 1982, 1988; Reder & Ritter, 1992). These studies alone call into serious question the typical description that at Age N, children are in Stage X, have rule X, have theory X, use strategy Y; at age N+n they are in stage Y, have rule Y, have theory Y, use strategy Y; and so on.

Adaptive Strategy Choices

Children's strategy choices have been shown to be adaptive in several ways. One involves their choice of whether to state a retrieved answer or to use a backup strategy (any approach other than retrieval, such as the sum and min strategies in addition). The more difficult a problem, the more often children use backup strategies to solve it. This pattern of strategy choice is adaptive because it enables children to use the faster retrieval approach most often on problems where it yields correct answers and to use the slower backup strategies most often on problems where they are necessary to produce accurate performance. Consistent with this analysis, forcing children to retrieve on all trials by imposing a very short time limit produces a sharp falloff in accuracy, with the falloff largest on the problems on
which children most often use backup strategies when they are allowed to choose freely (Siegler & Robinson, 1982).

This pattern of choices between retrieval and backup strategies has been found to be extremely general across populations and domains. It holds true with problem difficulty defined either by percentage of errors or by length of solution times; with high achieving and low achieving students; with suburban White and inner city Black children; and with addition, subtraction, multiplication, time telling, spelling, and word identification (Geary & Burlingham-Dubree, 1989; Geary & Wiley, 1991; Kerkman & Siegler, 1993; Lemaire & Siegler, 1995; Siegler, 1986, 1988a, 1988b).

Children also choose adaptively among alternative backup strategies. For example, when choosing between the min and sum strategies, children most often select the min strategy on problems where differences between addends are large and where the smaller addend is small (Siegler, 1987a). Thus, when solving single-digit addition problems, they are most likely to use the min strategy on problems such as $9 + 2$. Problems with large differences between addends are the ones on which the min strategy produces the greatest savings in amount of counting, relative to the main alternative approach, the sum strategy. Complementarily, the easiest problems on which to execute the min strategy correctly are ones on which the smaller addend is small. It makes sense for children to use the min strategy most often on problems where its advantages over alternative strategies are greatest.

Change

Four main changes in children’s strategy use occur with age and experience: acquisition of new strategies, changing frequency of use of existing strategies, improved efficiency of execution of strategies, and more adaptive choices among strategies (Lemaire & Siegler, 1995).

First consider acquisition of new strategies. Even after children know strategies that consistently yield correct performance on a class of problems, they continue to invent new strategies for solving them. For example, children who competently solve simple addition problems by using the sum strategy or retrieval go on to discover the min strategy (Siegler & Jenkins, 1989).

Above and beyond new strategies being added, there are substantial shifts in the relative frequency of strategies that are used to varying degrees throughout the period. For example, in 5- to 7-year-olds’ single-digit addition, three strategies tend to be present throughout the period: retrieval, the sum strategy, and guessing. Frequency of the first approach steadily increases from kindergarten to second grade, whereas that of the second and third steadily decreases (Siegler, 1987a).

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Changes also are evident in the skill with which each strategy is executed. With practice, children become both faster and more accurate in their execution of strategies. For example, in Siegler (1987a), mean time to execute the min strategy decreased from 6 sec among kindergartners to 4 sec among second graders, and percent correct when using the strategy increased from 71% to 93%.

Finally, with problem-solving experience, choices among strategies become increasingly adaptive. Examination of French second graders at three points during their learning of single-digit multiplication illustrated this point (Lemaire & Siegler, 1995). At the time of the first observation, in the first 10 days of the children’s learning, choices were already moderately attuned to problem difficulty. However, the correlations between problem difficulty and frequency of use of backup strategies were higher after 2 months of studying multiplication and still higher after four months of studying it.

Generalization

Observation of children in the weeks after they first discover a new strategy indicates that their generalization of it to new problems often takes a long time, at least when children possess other effective strategies (Kuhn, Schauble, & García-Mila, 1992; Schauble, 1990; Siegler & Jenkins, 1989). However, generalization of new strategies can be considerably hastened if children encounter conditions that highlight the new strategy’s advantages over previous approaches. For example, in a study of 4- and 5-year-olds’ discovery of the min strategy (Siegler & Jenkins, 1989), in the initial sessions after the discoveries, children used the new approach only occasionally. As long as typical single digit problems were being presented, even children who had given excellent explanations of the advantages of the new strategy (e.g., “You don’t have to count all those numbers”), continued more often to count from one. However, presentation of problems such as $3 + 22$, on which the min strategy worked far better than alternative approaches, led children who had previously discovered the strategy to use it much more often and then to generalize it to single-digit problems as well. Thus, although generalization of new strategies is usually slow, it can be facilitated if problems are presented on which alternative approaches do not work and on which the new strategy does.

Individual Differences

Recognizing the variability of strategy use within individuals raises the issue of whether strategy use also varies in interesting ways among individuals. Although research on broad cognitive styles has not identified many strong
consistencies in strategy use (Kogan, 1983; Sternberg, 1985), studies of more narrowly defined strategy choices have yielded more encouraging results. For example, examination of the choice between stating a retrieved answer and using a backup strategy has revealed consistent individual patterns in first graders' addition, subtraction, and word identification (Kerkman & Siegler, 1993; Siegler, 1988b). The research indicates three characteristic patterns: the good student, not-so-good student, and perfectionist patterns. Good students are children who usually rely on retrieval and who generally answer quickly and accurately. Not-so-good students sometimes use retrieval but generally execute both it and backup strategies quite slowly and inaccurately. Perfectionists are fairly fast and very accurate, but use retrieval even less often than the not-so-good students; instead, they rely heavily on backup strategies.

These patterns of individual differences in addition, subtraction, and word identification have been found in four different experiments, some involving high-SES, predominantly White, suburban populations and others involving low-SES, predominantly Black, inner-city populations. They also have proved predictive of standardized test scores and of future classroom placements: On the standardized tests, not-so-good students score significantly lower than the other two groups, are more likely to be classified as learning disabled, and are more likely to need to repeat a grade (Kerkman & Siegler, 1993; Siegler, 1988b). Thus, the differences between not-so-good students and the other two groups are of the type that are detected by standard psychometric tests. However, the differences between the strategy choices of good students and perfectionists are not apparent on these tests; they are more akin to differences in cognitive style.

Summary

The present perspective brings to center stage a different set of phenomena than are emphasized in traditional accounts of development. Rather than focusing on the problem solving approach that children use at each age, the perspective focuses on the set of approaches that children use. Highlighting this variability brings into the spotlight how children choose among the alternative approaches and what adaptive purposes the choices serve. The perspective also calls attention to several different types of changes: acquisition of new approaches, changes in the frequency of use of existing approaches, changes in the effectiveness with which these approaches are used, and increasingly adaptive choices among alternatives. The perspective also highlights how strategies are generalized beyond their initial contexts and how individuals vary in their strategy choices. We now describe a model aimed at accounting for these phenomena.

4. STRATEGY CHOICES ACROSS THE LIFE SPAN

ASCM: A MODEL OF STRATEGY CHOICE

Children's strategies first became a major topic of research in the mid-1960s (e.g., Flavell, Beach, & Chinsky, 1966; Keeney, Cannizzo, & Flavell, 1967). The early research documented large changes in memory strategies between ages 5 and 8 years. Five-year-olds were said to rarely use strategies such as rehearsal and organization; 8-year-olds were said to consistently use them. Especially interesting were the results of efforts to teach such strategies to 5- and 6-year-olds who did not spontaneously use them. Such children often learned the strategies, and their performance improved when they used them. Despite these benefits, the children usually did not continue to use the strategies later, even in quite similar situations.

This puzzle was an important impetus for initial thinking about children's strategy choices—the metacognitive approach. It was labeled "metacognitive" because it focused on how knowledge about cognition might control cognitive activities. The fundamental assumption was that young children's failure to use new strategies reflected their limited understanding of their own cognitive capacities and of why the new strategies were needed.

It is important to note that the metacognitive models that were originally proposed focused on explicit, rationally derived, conscious metacognitive knowledge. The term metacognitive also at times has been used to refer to processes that are implicit, not derived from rational consideration, and unconscious. Greeno, Riley, and Gelman's (1984) planning networks, VanLehn's (1983) repair models, and Halford's (1995) mental models, like our own model of strategy choice, fit this category. In the present context, however, the term metacognitive is used only in its original sense of explicit, rationally derived, conscious knowledge.

Metacognitive approaches assume that strategy choices are made through the cognitive system's explicit knowledge of its own workings. This knowledge is often said to be used by an "executive processor," which decides what the cognitive system should do (Case, 1978; Kluwe, 1982; Sternberg, 1985). Schneider and Pressley (1989) described the executive processor as follows:

This executive is aware of the system's capacity limits and strategies. The executive can analyze new problems and select appropriate strategies and attempt solutions. Very importantly, the executive monitors the success or failure of ongoing performance, deciding which strategies to continue and which to replace with potentially more effective and appropriate routines. In addition, the efficient executive knows when one knows and when one does not know, an important requirement for competent learning. (p. 91)

Such metacognitive approaches are useful for pointing to one way in which, in principle, intelligent strategy choices could be generated. How-
ever, they also have a number of weaknesses, both theoretical and empirical (Brown & Reeve, 1986; Cavanaugh & Perlmutter, 1982; Siegler, 1988a). As statements of theory, they are also too reminiscent of the homunculus that sits atop the rest of the cognitive system and tells it what to do. Not unrelated to this problem, such metacognitive approaches have been vague about the mechanisms that produce the phenomena of interest. Do people make explicit judgments about their intellectual capacities, available strategies, and task demands every time they face a task they could perform in multiple ways? If not, how do they decide when to do so? Do they consider every strategy they could use on the task or only some of them? If only a subset, how do they decide which ones? How do people know what their cognitive capacity will be on a novel task or what strategies they could apply to it? The apparent simplicity of the metacognitive models masks a world of complexity.

Empirical evidence has raised questions about the fundamental assumption that underlies the models. Relations between explicit, verbalizable metacognitive knowledge and cognitive activity have proven weaker than originally expected (Cavanaugh & Perlmutter, 1982; Schneider 1985; Schneider & Pressley, 1989). This has cast doubt on whether explicit metacognitive knowledge in fact plays a central role in children’s strategy choices.

Consider an empirical study of the role of metacognitive knowledge in children’s arithmetic. The implicit causal pathway to strategy choices suggested by metacognitive models is:

\[ \text{Problem difficulty} \rightarrow \text{Metacognitive judgments of problem difficulty} \rightarrow \text{Strategy choices} \]

To test whether children choose arithmetic strategies in this manner, Siegler and Shrager (1984) obtained measures of each of the three constructs. Problem difficulty was assessed in terms of children’s percent correct on each of 25 simple addition problems. Metacognitive judgments of problem difficulty were obtained by asking the same children whether each problem was very hard (2 points), kind of hard (1 point), or easy (0 points). Strategy use was assessed by trial-by-trial analyses of videotapes of each child solving each addition problem.

The results indicated some relation between the children’s metacognitive judgments and the other two measures. Actual difficulty of each problem correlated $r = .47$ with children’s judgments of the difficulty of the problem. Judgments of the difficulty of each problem correlated $r = .51$ with percentage of trials on which children used backup strategies on the problem. Both of these correlations were significant. However, they were not nearly strong enough to account for the very strong correlation ($r = .91$) between actual difficulty of each problem and strategy choices on the problem. Thus, young children’s metacognitive knowledge might contribute to their extremely adaptive strategy choices, but could not account for them. This perspective motivated development of a series of models that demonstrated how highly adaptive strategy choices could arise out of implicit strategy choice mechanisms (Siegler, 1987b, 1988b; Siegler & Shrager, 1984). The most advanced of these is ASCM, a model described more fully in Siegler and Shipley (1995). The particulars of the model concern development of single-digit addition skills between ages 4 and young adulthood, but its basic structure is intended to model strategy choices more generally.

The Model’s Basic Structure

Figure 4.1 illustrates ASCM’s overall organization. Strategies operate on problems to generate answers. The solution process yields information not only about the answer to the particular problem (e.g., 3 + 5), but also about the time required to solve the problem using that strategy and the accuracy of the strategy in answering the problem. This information is used to modify the database regarding the strategy, the problem, and their interaction.

**The Database.** The type of information that gets entered into the database is illustrated in Fig. 4.2. Through their experience solving problems, children gain knowledge of both strategies and problems. Knowledge of each strategy can be divided into knowledge based on actual data and knowledge based on projections (inferences) from that data.

The actual database includes global data—that is, data about each strategy’s past speed and accuracy aggregated over all problems that the system

![Fig. 4.1. An overview of the organization of ASCM.](image-url)
Organization of database

Strategies

S1

\[
\begin{array}{c}
\text{Actual} \\
\text{Projected} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Accuracy} \\
\text{Novelty} \\
\text{Speed} \\
\end{array}
\]

S2

\[
\begin{array}{c}
\text{Actual} \\
\text{Projected} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Accuracy} \\
\text{Novelty} \\
\text{Speed} \\
\end{array}
\]

Problems

P1

A1 A2 A3

P2

A1 A2 A3

FIG. 4.2. The organization of ASCM's database in which information about past performance is linked to the strategy that produced the performance and the problem on which the performance occurred.

Inclusion of the novelty data was motivated by an attempt to answer the question: How do new strategies ever come to be used in situations in which existing strategies work well? In the context of young children's addition, if a child can consistently solve a problem by using the sum strategy, why would the child ever try the min strategy on the problem? ASCM deals with this issue by assigning novelty points to newly discovered strategies. These novelty points temporarily add to the strength of new strategies, and thus allow the new strategies to be tried even when they have little or no track record.

With each use of a new strategy, some of its novelty strength is lost, but information about its speed and accuracy is gained. This leads to the strategy's probability of use increasingly being determined by the database on its effectiveness. The idea of novelty being a kind of strength was suggested by the observation that people (especially children) are often interested in exercising newly acquired cognitive capabilities (Piaget, 1970) and by the realization that without a track record, a newly acquired strategy might never be chosen, especially if reasonably effective alternatives were available.

Whenever ASCM is presented a problem, it uses these speed, accuracy, and novelty data to project how well each strategy is likely to do in solving the problem. If a strategy has never been used on a particular problem, ASCM's projections are based solely on global and featural data. If the strategy has never been used on the particular problem or on any problem with that feature, only global data are used to derive the projection.

The Model's Operation. ASCM is implemented as a running computer simulation. Its working can be summarized as follows:

1. At the beginning of its run, ASCM knows only the two addition strategies that are common among 4-year-olds—retrieval and the sum strategy—and basic procedures for choosing strategies, collecting data on the outcomes they generate, and projecting their future usefulness. These latter competencies are hypothesized to be basic properties of the human information processing system and to be present from birth.

2. During the learning phase, the simulation is repeatedly presented the 81 basic addition facts formed by all possible combinations of addend values 1–9 inclusive. The problems are presented equally often. In the absence of data on presentation rates over the large age range being modeled, this seemed the most conservative assumption.

3. After a number of exposures to each problem (60 trials/problem in the simulation runs reported here), the min strategy is added to those initially available. This is done to correspond to the time, usually sometime during first grade, when children add the min strategy to their repertoire. The process of strategy discovery is not yet modeled.
4. Strategy choices are based on the projected strength of each strategy. Projected strength is a function of the strategy's past speed and accuracy on problems as a whole, on problems with features in common with the current one, and on the particular problem being solved. For new strategies, the strategy's novelty boosts its strength beyond what its past performance alone would justify.

5. A logistic equation weights these sources of data according to the amount of information they reflect. When a strategy has rarely been used on a particular problem, global and featural data are weighted most heavily. As more information becomes available about how well the strategy works on the problem, problem-specific information receives increasing weight, eventually exercising the largest influence. The reasoning is that data derived from a few uses of a strategy are inherently noisy, but that problem-specific information based on a substantial data base ultimately is the best predictor of a strategy's future effectiveness on that problem. A similar logistic equation is used to weight data according to how recently they were generated. The reasoning underlying this decision was similar: Recent performance on a problem is given greater weight because it is likely to better predict future performance on the problem.

6. Each time a problem is presented, these weighted sources of information provide the input to a stepwise regression equation, which computes the projected strength of each strategy on the problem.

7. Probability of choosing a particular strategy is proportional to that strategy's projected strength relative to that of all strategies combined. The simulation attempts to execute whichever strategy is chosen.

8. If retrieval is chosen, three variables determine whether it can be executed to completion: the confidence criterion, the search length, and the associative strength of the retrieved answer. The confidence criterion is a threshold indicating how sure the child must be to state an answer. The search length indicates the number of efforts the child will make before ending attempts at retrieval. Both the confidence criterion and the search length vary randomly from trial to trial—they are associated neither with the strategy nor with the problem.

The associative strength of each answer, incorrect as well as correct, reflects the frequency with which that answer has been generated in the past (which in turn is a function of the frequency with which the problem has been presented, the probability of generation of each answer in the past, and interference from related problems; see Siegler and Shrager, 1984, for a detailed account of how this retrieval mechanism operates). The probability of each answer being retrieved is proportional to its associative strength relative to the associative strengths of all of the answers. If the associative strength of the retrieved answer exceeds the confidence criterion, that answer is stated; if not, but the search length has not been reached, retrieval is attempted again.

9. If retrieval does not yield a stable answer in the allotted number of searches, the model returns to the strategy choice phase and chooses among the backup strategies. The process is the same as at the beginning of the trial, except that retrieval is excluded from consideration (because it has already been tried). Thus, the probability of a given backup strategy being chosen at this point reflects its projected strength relative to that of all backup strategies.

10. Any strategy can generate either a correct answer or an error. Probabilities of errors using the sum and min strategies are proportional to the number of counts required to execute the strategy on that problem. The errors arise through double counting or skipping an object in the representation of the problem. Each count entails a probability of error; thus, the greater the number of counts, the more likely that an error will be made. On retrieval trials, errors arise through an incorrect answer being retrieved and having sufficient associative strength to be stated.

11. Solution times on backup strategy trials are proportional to a constant multiplied by the number of counts that are executed. The constant is smaller for the sum than for the min strategy, because children take less time per count in counting from one than in counting from other numbers (Siegler, 1987a). Solution times on retrieval trials reflect a constant multiplied by the number of searches prior to locating a stable answer. This constant is much smaller than those used with the backup strategies, reflecting the fact that retrieval is much faster than the sum or min strategies.

12. Each time an answer is advanced, ASCM increases the association between that answer and the problem and adds information regarding the speed and accuracy with which the answer was generated to the database for the strategy.

13. Each execution of a backup strategy also brings an increase in the strategy's speed and a decrease in its probability of generating an error. Thus, strategy execution improves with practice.

The Model's Performance

The tests of ASCM's performance involved a learning phase of a given length and a test phase that indicated the performance generated by the simulation after that amount of experience. During the learning phase, performance on each trial altered the data base regarding strategies, problems, and answers. The analogy was to experience that children would have had prior to entering the experimental situation. During the test phase, in contrast,
the database remained constant. The analogy was to children's performance in the experimental situation, after their having had a given amount of preexperimental experience.

ASCM can be understood in terms of its treatment of the five basic phenomena described earlier: variability, choice, change, generalization, and individual differences.

Variability. ASCM uses diverse strategies both within and between problems. It tends to use strategies most often on the problems where they work best, but strategies that work less well also are sometimes used.

Adaptive Strategy Choices. ASCM generates strong correlations (r's > .90) between problem difficulty and frequency of backup strategy use on the problem. This is true both when problem difficulty is measured by percent errors on each problem and when problem difficulty is measured by mean solution time on the problem. The model's pattern of choices also is much like those of children. Siegler and Shipley (1995) correlated ASCM's choices with those of 129 children who were finishing first grade. After 750 trials/problem, the simulation's percent use of backup strategies on each problem correlated r = .93 with that of the children. The best predictors of its performance, size of the smaller addend and differences between its addends, also were the same as those of the children.

Change. Perhaps the single most essential property of a simulation of acquisition of arithmetic knowledge is that it progress from the relatively inaccurate performance characteristic of children just beginning to add to the virtually perfect performance that characterizes children by fifth or sixth grade. ASCM met this test; after a learning phase of 60 trials/problem, its performance was 31% correct, whereas after a learning phase of 1,250 trials/problem, its performance was 99% correct.

As shown in Fig. 4.3, ASCM's changes in frequency of use of the three strategies also paralleled children's. At first, the simulation used only the sum strategy and retrieval (the only strategies it knew), with the sum strategy being employed on the large majority of trials. After the min strategy was added, it became the most frequently used strategy, with the sum strategy and retrieval also being used on substantial numbers of trials. This corresponds to children's performance at around first grade. Beyond this point, use of both the sum and the min strategies decreased, and retrieval became increasingly dominant. After a learning phase of 1,250 trials/problem, retrieval was used on 95% of trials.

A third key type of progress involved decreases in solution times. The overall decrease in times was produced by two factors: shifts from the slower backup strategies to the faster retrieval approach, and faster execution of each strategy. More interestingly, it went through the same sequence of variables that best predicted mean solution times. With short learning phases, its mean solution time on each of the 81 problems, like those of children, were best predicted by the sum of the addends on that problem (because the sum strategy was used most often). With longer learning phases, its mean time on each problem, again like that of children, was best predicted by the difference between the addends and the size of the smaller addend on the problem (because the min strategy was used most often). With still longer learning phases, its mean time on each problem, again like that of children, was best predicted by the product of the addends (a predictor believed to reflect use of retrieval; see Geary, 1994).

Finally, parallel to the results of Lemaire and Siegler (1995), the adaptiveness of ASCM's choices increased over trials until ceiling effects on accuracy and floor effects on percent use of backup strategies began to reduce the correlations at the end of its run. In particular, the correlations between percent of backup strategy use on each problem and percent errors steadily increased with increasing experience solving the problems. The correlations between percent backup strategy use on each problem and mean solution time on the problem showed similar increases with practice.
**Generalization.** As noted earlier, a key requirement for a realistic model of arithmetic is that it be able to generalize its knowledge of strategies to new problems. To test ASCM's ability to generalize, Siegler and Shipley (1995) presented it with 50 learning-phase exposures to each of 10 single-digit problems and then, during the test phase, examined its strategy choices on the other 71 problems. The procedure was repeated with 500 learning-phase trials/problem for each of the 10 problems, to see if generalization improved with experience. We were particularly interested in whether ASCM would choose the min strategy most often on problems where it was easiest to execute (i.e., where it could be executed with the fewest counts), where its advantage in reduced counting over the sum strategy was greatest, or both. For example, we wanted to know whether it would choose the min strategy especially often on 9 + 1, a problem that has both of these properties.

ASCM showed exactly this pattern of generalization. On the 71 problems that had not been presented in the learning phase, the best predictor of percentage of trials on which the min strategy was used was the difference between the addends. The differentiation between problems on which the min strategy was more and less helpful increased with learning; differences between the addends was a better predictor of amount of generalization of the min strategy after a learning phase of 500 trials/problem than after one of 50 trials/problem. This made sense, because the model was obtaining increasingly valid data about each strategy's effectiveness on different problems.

**Individual Differences.** As noted earlier, Siegler (1988b) found substantial individual differences in children's approaches to addition, subtraction, and word identification tasks. The children could be classified into three distinct groups: the good students, the not-so-good students, and the perfectionists. As the names suggest, the good students were both more accurate and more likely to retrieve answers than the not-so-good students. The perfectionists were as accurate as the good students, but used retrieval even less often than the not-so-good students.

ASCM suggested a simple means through which the three individual difference groups could arise: parametric variation in peakedness of the distributions of associations and in confidence criteria. **Peakedness** refers to the degree to which the associative strength of answers to a problem is concentrated in a single answer (ordinarily the correct one). The more peaked the distribution of associations, the more likely that the correct answer will be retrieved, and the more likely that its associative strength will exceed the confidence criterion, leading to statement of the answer. The confidence criterion is a kind of threshold; the higher it is, the greater the associative strength of the retrieved answer must be to exceed it, and therefore, the fewer retrieved answers will be stated.

This view suggests an interpretation of each group's pattern of performance. The good-student pattern would arise from a combination of peaked distributions of associations and a wide range of confidence criteria. This would lead to both high accuracy and frequent use of retrieval, because the highly peaked distributions would result in frequent retrieval of the correct answer, which would have enough associative strength to exceed most confidence criteria and therefore be stated. The not-so-good-student pattern would arise from flat distributions of associations and low confidence criteria. This would generate inaccurate performance and medium amounts of retrieval, because incorrect answers would often be retrieved and would sometimes have sufficient associative strength to exceed the low confidence criteria and be stated. The perfectionist pattern would arise from peaked distributions and very high confidence criteria. This would lead to accurate performance but to low amounts of retrieval, because only the most peaked distributions would have correct answers with enough associative strength to exceed the very high confidence criteria.

To test this interpretation, Siegler and Shipley (1995) created three variants of the simulation. They differed only in their values of the two parameters hypothesized to underlie the individual differences: probability of correct execution of backup strategies (which largely determines the peakedness of distributions that are formed) and range of confidence criteria. The simulation of the not-so-good students' performance executed backup strategies less accurately than did the simulations of the perfectionists' and good students' performance, which were identical to each other in accuracy of execution of these strategies. The confidence criteria of the not-so-good students were consistently low (.10–.50), those of the perfectionists consistently high (.50–.90), and those of the good students included both low and high values (.10–.90). Other than these two parameter values, the simulations of the three groups were identical.

The variations in these two parametric values were sufficient to account for the observed pattern of individual differences. ASCM's simulation of the not-so-good students produced lower percentages correct than its simulations of the good students and perfectionists, which did not differ. Also as with children, ASCM's "good student" simulation produced the greatest amount of retrieval, its "not-so-good student" simulation the next most, and its "perfectionist" simulation the least. The simulations thus illustrated how qualitatively different patterns of performance can arise through parametric variations within the same basic processing system.

**Implications**

Within ASCM, the capacity to generate highly adaptive strategy choices is viewed as a basic part of human cognition. All mechanisms within it are believed to be present from extremely early in development and to remain present throughout the life span. This view implies that the types of adap-
tive strategy choices described earlier also should be present throughout the life span. In the next two sections, we describe data that share striking parallels with children's addition: variable strategy use, adaptive strategy choice, gradual change, generalization beyond the particular problems that have been encountered, and individual differences in style as well as competence. The first section focuses on the strategy choices of infants, the second on the strategy choices of old people.

HOW INFANTS SELECT STRATEGIES FOR ASCENDING AND DESCENDING SLOPES

Infants' first strategies are geared toward homely, everyday problems—resisting gravity, reaching and grasping, exploring objects, and locomoting independently around the environment. Traditionally, researchers described the development of motor skills as a progression of stages, in which infants discarded less mature strategies for more mature ones at each stage (e.g., Gesell & Thompson, 1938; McGraw, 1945). For example, when a small sugar pellet was placed in front of infants of varying ages, the youngest infants were reported to rake at it with their hands, older babies to scoop it into their hands, and still older ones to use a pincer grip to grasp it between thumb and forefinger. Younger infants were reported to begin crawling by pushing backward, older babies to inch forward on their bellies, even older infants to crawl on hands and knees, and still older ones to crawl on hands and feet.

More recently, researchers have begun to analyze infants' strategies in greater detail (e.g., Adolph, Eppler, & Gibson, 1993b; Gibson, 1988; Thelen, 1995). They have found greater variability in infants' repertoires at a single age or developmental level than previously indicated (e.g., Goldfield, 1994). For example, infants often reach unimanually on one trial and bimanually on the next for the same size toy (Corbett & Thelen, in press). Babies often use multiple patterns of interlimb coordination to crawl a few feet along a flat path, even demonstrating variability from cycle to cycle (Viereck, 1995). Such observations of variability raise the question of whether young infants select strategies adaptively to cope with practical motor problems.

Investigators working within a very different functional framework—the ecological approach (J. J. Gibson, 1979)—have found that older children and adults precisely match their action strategies to the changing exigencies of the physical environment. For example, adults and school children switch from walking on two feet to climbing on all fours to ascend stairs with risers higher than a constant proportion of their leg lengths (Mark, 1987; Pufall, 1992; Warren, 1984). Adults shift from walking head on to turning their bodies sideways to pass through apertures narrower than a constant proportion of their shoulder widths (Warren & Whang, 1987). Preschoolers adjust their speed and step length to cope with walking over balance beams of various widths and heights (Adolph, Ruff, Cappazolli, & Kim, 1994). To examine whether very young infants also select locomotor strategies adaptively, Adolph and colleagues challenged babies with a novel locomotor task—going up and down steep and shallow slopes (Adolph, 1995; Adolph, Eppler, & Gibson, 1993a).

The Slope Task

Slopes are an ideal venue for studying infants' strategy choices for two reasons. First, slopes are novel, allowing firsthand observation of how infants learn to cope with them. Most babies have few if any opportunities to learn about slanted surfaces in everyday situations. Parents typically limit children's exposure to ascent and descent throughout their first year of life by gating household stairs and closely monitoring their babies after the babies discover how to clamber on and off furniture. Second, because ability to locomote successfully on slopes depends on their steepness and on whether the goal is to go up or down, the task allows straightforward measurement of the degree of adaptiveness of the strategy choices. The role of steepness is obvious, but the effects of direction of movement may require some explanation. Biomechanically, going uphill is easier than going down, because infants can support their weight on a fully extended limb, and their hands are in a good position to stop a fall. Crawling and walking downhill require infants to support body weight and maintain balance on a bent arm or leg as the other limb swings forward, necessitating more strength and control as infants resist the pull of gravity. Both crawling and walking downhill place infants' hands in an awkward position to break a fall, thus adding to the difficulty of controlling the movement (Adolph, 1995; Adolph et al., 1993a).

A Study of Infants' Locomotor Strategy Choices. Adolph (1995) examined locomotor strategy choices in 14-month-old walking infants. Toddlers had different amounts of locomotor experience (ranging from 10 to 137 days) and skill, but all could walk at least 10 steps independently. The basic plan was to compare infants' strategy choices on safe slopes, where walking was possible, with their strategy choices on risky (i.e., steep) slopes, where they needed to use an alternative method of locomotion to avoid falling.

Materials and Procedure. Infants were tested on a large, carpeted walkway with a slope that could be set at angles ranging from 0° to 36° in 2° increments. The shallowest hills were safe for all babies; the steeper hills were increasingly risky. Each trial began with babies perched upright on the starting platform, facing the slope from their typical vantage point.
Parents stood at the far end of the walkway urging their babies to come up or down, offering Cheerios and attractive toys as enticements. An experimenter followed alongside the infants to ensure their safety.

In each test session, the experimenter used a psychophysical staircase procedure to identify the boundary between safe and risky slopes (described in Adolph, 1995). Slope boundaries were the steepest hills on which infants could walk up or down. These slope boundaries provided an index of the infants' physical abilities, as well as a way to compare strategy choices across infants relative to each baby's current level of walking skill.

The experimenter coded each trial online as success (walked safely), failure (tried to walk but fell), or refusal (slid down, climbed up, or avoided going). For the purpose of estimating slope boundaries, failures and refusals were treated as equivalent, unsuccessful outcomes. Success on a trial led to presentation of a steeper slope on the next trial; failure or refusal led to presentation of a shallower slope. The process continued until the experimenter identified a slope boundary according to a 67% criterion (the steepest hill on which the infant walked successfully on at least 3/4 of trials and failed or refused on at least 3/4 of trials at the next 2° increment and all steeper hills). To maintain infants' interest, easy baseline slopes (0° to 6°) were interspersed with the more challenging ones.

There was a wide range in infants' abilities. Some toddlers could manage only very shallow hills; others could walk over terrifically steep ones. Infants who were more proficient walking on flat ground (indexed by the length and width of their steps) and who had more days of walking experience were also more skillful walking up and down slopes, supporting the reliability of estimates of slope boundaries. As in earlier research (Adolph et al., 1998a), infants had steeper boundaries walking uphill than downhill (Fig. 4.4). Together, these results mean that a safe hill for a proficient walker is risky for a less skilled child and that hills safe for ascent are risky when going down.

The "GO Ratio." The critical question was whether infants' adapted their strategy choices to their current walking prowess on slopes. Infants' typical method of locomotion for travel over flat ground (walking) could be viewed as a sort of backup or default strategy. Presumably, if the surface appeared tractable, infants should walk up or down the hill, without shifting from the position in which the experimenter placed them on the starting platform. On the other hand, if infants decided the slope was risky, they should resort to a more effortful backup strategy: going up or down on all fours; sliding down head first, prone like Superman; sliding down while sitting on their bottoms; or scooting down on their bellies, feet first. A final backup strategy was to avoid going, which presumably indicated that they considered the slope too steep for either their typical method or available alternatives.
the boundary. Maximally adaptive strategy choices would imply two patterns: (a) a high GO ratio on safe hills—that is, ones shallower than the slope boundary, where probability of success is high; and (b) a low GO ratio on risky hills—that is, ones steeper than the slope boundary, where probability of falling is relatively high. Infants might err on the side of caution with a low ratio on perfectly safe slopes, or they might err on the side of boldness with a high ratio on hills too steep for them to traverse safely by walking.

Variability

On uphill trials, babies had few options—walking, crawling up on all fours, or avoiding. Variability in strategy choice for ascent was limited to only two of the three options. Infants always walked up safe hills. On risky hills, all 31 babies attempted their typical walking strategy on some trials, and most infants (24) also tried to climb up on hands and knees. Toddlers never refused to go at all on uphill trials. Similarly, in an earlier experiment with 14-month-old walkers tested on 10°, 20°, 30°, and 40° slopes, nearly all toddlers walked up shallower slopes, most crawled up steeper ones, and only 2 babies avoided ascent (Adolph et al., 1993a). Eight- to 9-month-old crawling infants pitted against the same four upward slopes showed similar compression of variability. They all attempted upward slopes on hands and knees and only 3 of 28 crawlers avoided ascent.

Babies had several options for descending slopes: walking, crawling, sliding prone, sitting, backing, or avoiding. Strategic variability was much greater on downhill trials, particularly downhill trials on risky slopes. Toddlers used a mean of 1.03 strategies for descending safe hills versus a mean of 2.28 strategies for descending risky hills. On safe hills, 30 toddlers used solely their typical walking strategy and only 1 baby walked and also crawled. Although any single backup strategy would have been sufficient to descend on risky hills, many infants used multiple methods. On risky hills, 1 infant used 3 backup strategies, 10 toddlers used 2 backup strategies, and 14 babies used only 1 backup strategy (2 reckless infants never used backup strategies and instead attempted to walk down every slope). Sliding in a sitting position and backing down feet first were used most often, followed by crawling, sliding prone and refusing to go at all. Variability was even more impressive in Adolph et al.’s (1993a) study where toddlers had only 4 descending trials in total, one trial each at 10°, 20°, 30°, and 40°. Eleven toddlers used 2 backup strategies, 10 used only 1 backup method and 2 intransigent infants attempted to walk down every hill. Eighteen toddlers used a backup sliding strategy rather than avoidance on at least one of the four descent trials. Variability was sharply reduced in the sample of 8.5-month-old crawling infants. Although every infant demonstrated the physical ability to execute all of the sliding positions, the only backup strategy used by crawlers was avoidance (14 Ss). As predicted by ASCM, there were changes in frequency of strategy use with age and locomotor experience, but no stage-like transitions from one strategy to another.

Adaptive Strategy Choices

The most stringent test of the adaptiveness of strategy choices was the difference between GO ratios on safe and risky hills. The ideal pattern of judgments would be high GO ratios on safe hills, where probability of successful walking was high, and low GO ratios on risky hills, where probability of successful walking was low. In addition, to the degree that choices were adaptive, the difference between GO ratios on safe and risky hills was expected to be higher on descent trials. On such trials, falling was relatively aversive, and the experimenter had to rescue infants to prevent injury. In contrast, on ascent trials, failures were relatively inconsequential, because the babies could catch themselves if they started to fall.

Consistent with this perspective, GO ratios were high on safe slopes and decreased steadily on risky hills, from .99 to .23 for uphill and from .94 to .11 for downhill (see top panel of Fig. 4.5). At each slope increment steeper than toddlers’ slope boundaries, GO ratios were nearly twice as high on ascent trials as on descent ones. Toddlers’ strategy choices were remarkably consistent. If they refused to walk over a particular slope increment, they also slid or avoided all steeper increments.

![Graph showing GO ratios adapted to slopes steeper and shallower than slope boundary](image-url)

**FIG. 4.5.** GO ratios adapted to slopes steeper and shallower than slope boundary. GO ratios are proportion of trials where infants attempted to walk. Slope boundaries are steepest hills on which infants can walk without falling on at least 3% of trials.
On ascent trials, toddlers’ strategy choices (walking versus climbing on all fours) were based on a brief glance before starting up. As in earlier research (Adolph et al., 1993a), infants hurled themselves forward onto upward slopes, trial after trial, with virtually no hesitation or prior haptic exploration. The story was very different on descent trials. As in the previous study, infants were more wary about going down. Exploratory activity on the starting platform precisely mirrored infants’ perceptual judgments. Infants hesitated, looked, touched, and tested alternative sliding positions before starting down hills on which they used backup strategies (see bottom panel of Fig. 4.5). The evidence suggests that adaptive strategy choice is an essential and pervasive aspect of learning perceptual-motor skills (e.g., Adolph et al., 1993b; Gibson, 1979), as well as skills traditionally viewed as cognitive.

Change

Results of Adolph and colleagues’ studies (Adolph, 1995; Adolph et al., 1993a) indicated the endpoints of an interesting developmental progression. Fourteen-month-old walking infants in these studies carefully adapted their locomotor strategies to the slant of steep and shallow slopes. They walked down safe hills within the limits of their physical abilities and used backup strategies on risky hills steeper than their walking abilities. In contrast, the strategy choices of younger crawling infants, 8.5-month-olds, were indiscriminate. More than half of the crawlers plunged headlong down every hill, falling on consecutive trials at 10°, 20°, 30°, and 40°, requiring rescue by the experimenter.

A recent longitudinal study tracking infants from their very first weeks of crawling until months after they began walking (Adolph, 1993) indicates that the increase in adaptive strategy choices from crawling to walking was not due to a stage-like transition resulting from maturation (the walkers in both studies were 6 months older than the crawlers), or the increased balance requirements of standing on two feet after walking onset (walking is more tipsy than crawling). Instead, change was due to everyday experience traveling around in infants typical method of locomotion. When infants first began crawling, regardless of age, they behaved like the crawlers in Adolph et al.’s (1993a) study. Most attempted risky hills and fell headlong. Strategy choices became increasingly adaptive over weeks of crawling until, by their last week of crawling, judgments were nearly perfect. This means that infants made sound and adaptive judgments for coping with slopes, despite continuous change in their crawling abilities on slopes. Surprisingly, there was no transfer from crawling to walking. In their first week of walking, infants attempted to walk down hills far beyond their capabilities. Again, errors decreased steadily as infants gained experience with that type of movement and posture. By 14 months of age, most infants had several weeks of walking experience and their GO ratios were comparable to the infants in the Adolph (1995) study.

ASCM predicts that when people have little or no local and featural information, their strategy choices are made primarily on the basis of global information about each strategy. Consistent with this prediction, new crawlers and walkers were at first biased toward their typical method of locomotion but with weeks of experience, they became less biased in this direction. Early in their experience, they relied heavily on their default locomotor strategy (crawling or walking) on all trials, regardless of whether they were ascending or descending and regardless of the slope of the hill. Over weeks of crawling and walking, infants increasingly often used a backup strategy for coping with the risky downhill slopes, while continuing to rely on their usual locomotor approach on hills where it was effective.

Generalization

Specific experiences with descent were not related to toddlers’ GO ratios or use of backup strategies (Adolph, 1995). There were no differences between infants encountering slopes for the first time and babies who had experience going down playground slides or household stairs. Ten children had gone down a small playground slide independently, but their GO ratios were distributed in the same patterns as children without prior experience descending steep slopes. Fourteen toddlers had gone down a short flight of stairs independently using the backing strategy; only 10 of these infants used the backing strategy to descend slopes, and 6 infants without experience backing down stairs used this strategy spontaneously to cope with slopes. Three children had experienced falls resulting in serious injuries while attempting to go down stairs, but they were no more cautious on slopes than children who had never experienced a serious fall. Similarly, of the 12 toddlers reported to back down stairs in Adolph et al.’s (1993a) study, only 8 backed down slopes. Of the 11 toddlers who had never backed down stairs, 4 babies used this strategy for the first time on slopes.

Even more striking evidence of generalization from everyday locomotor experience to strategy choice in the novel slope task was provided by Adolph’s (1993) recent longitudinal study. Adolph included a control group to assess effects of practice in the slope task. Slope boundaries and GO ratios were similar between control babies tested only three times on slopes and experimental infants tested every three weeks on slopes. Both groups improved, but the rates of improvement and the absolute levels of performance were similar. The data suggest that learning resulted from everyday locomotor experience in infants’ homes and play yards, rather than from experience in the experimental situation.
Individual Differences

Toddlers’ behavior in the slope task revealed consistent individual differences, suggesting differences in their confidence criteria as well as differences in their ability to discriminate safe from risky hills (Fig. 4.6). Seventeen of the 31 14-month-olds in Adolph (1995) walked down safe hills and slid down nearly all risky ones (Group A). Their GO ratios averaged .90 at the slope boundary, .10 on slightly steeper hills (2–8 degrees beyond the boundary), and 0 on all steeper slopes. Among these 17 children, 7 showed perfect step functions, walking on all hills with less than a certain slope, using backup strategies on all steeper ones. These children’s data indicate that they underestimated their abilities; they used backup strategies even on hills that they might have been able to traverse successfully using their usual locomotor approach. Like the “perfectionists” solving addition problems, they may have set extremely high confidence criteria for walking down the hill. The other 10 babies in the subgroup of 17 behaved like Siegler’s (1988b) “good students.” Their thresholds for use of a default strategy were high enough to thwart serious errors, but low enough that they pushed their outer window of possibilities for learning.

Two of the 31 children in the cross-sectional sample exhibited slightly more lenient response criteria (Group B). Their GO ratios were high until they encountered slopes of intermediate difficulty (10–16 degrees steeper than their boundaries), where ratios decreased sharply and remained at 0. Eight additional children seemed to have an extremely lenient response criteria (Group C). Their GO ratios were near 1.0 until they encountered impossibly steep hills (at least 18 degrees steeper than boundary). Together these 10 babies resembled the “not-so-good students” described in Siegler (1988b), in that they seemed to set very low confidence criteria and therefore to use their usual approach even where it was unlikely to succeed.

The remaining four infants in this cross-sectional sample responded indiscriminately (Group D). Their GO ratios never decreased below .50 on any slope. Like the two babies in the Adolph et al. (1993a) sample, they behaved as though they had no fear of hills. Their recklessness may reflect confidence that the experimenter would rescue them (they were indeed caught on every risky trial!), or it may reflect more serious difficulties in perceptual-motor learning. In summary, it does not take long for individual differences in strategy choices to become apparent; they can be seen even around children’s first birthdays.

STRATEGY CHOICES OF YOUNG AND OLD ADULTS

In many situations, the strategies used by older adults differ considerably from those used by younger ones. This has been documented in such varied domains as memory strategies (Cimbalo & Brink, 1982), searching for

Much less is known, however, about the adaptiveness of the choices that old people make. The only two studies of which we are aware (Geary, Frensch, & Wiley, 1995; Geary & Wiley, 1991) reported that old people's frequency of use of backup strategies in addition and subtraction vary significantly with characteristics of the problems, in particular with the sum of the addends and the size of the number being subtracted. In the case of addition, the magnitude of the correlation ($r = .80$) was comparable to those typically observed with children and younger adults; in the case of subtraction, the magnitude was much weaker ($r = .37$). Thus, it was unclear whether elderly adults choose strategies as adaptively as children and younger adults.

To compare as precisely as possible the adaptiveness of the strategy choices of younger and older adults, Siegler and Lemaire (in press) developed a new method for assessing adaptiveness. Past studies of strategy choices have measured adaptiveness of choices primarily by relating percent use of a given type of strategy (e.g., addition) to problem difficulty, usually as measured by percent errors or mean solution time on each problem. Usually, these estimates of problem difficulty have been obtained from performance in choice conditions, in which subjects could use any strategy they chose. This method works well for most purposes, but can artificially inflate or deflate the apparent adaptiveness of the choices. For example, if people are more accurate in using backup strategies than retrieval, the method will artificially deflate the correlation obtained (because subjects will compensate for the differences in problem difficulty by using the more accurate backup strategies more often on more difficult problems).

The other method that sometimes has been used is to estimate problem difficulty solely from performance when only retrieval is allowed. This avoids the aforementioned confounding, but leads to problem difficulty being estimated solely by difficulty when retrieval is used. If problem difficulty differs when different strategies are used, then this procedure yields an incomplete picture of problem difficulty when multiple strategies are available.

Thinking about how ASCM works suggested a promising alternative approach to measuring the adaptiveness of strategy choices, the choice/no-choice design. Recall that within ASCM, predicted use of each strategy is a function of the strength of that strategy relative to that of alternative strategies. This implies that use of a given strategy should be a function of the difference between the strengths of the strategies. To obtain estimates of this difference requires determining how difficult each problem would be using each available strategy, and then testing how well their choices are predicted by the differences between or among the difficulties.

Siegler and Lemaire (in press) used this method in two experiments. In the two-strategy experiment, the choice being studied was whether to solve multiplication problems using a calculator or via mental arithmetic. Both college students and elderly adults made these choices. In the three-strategy experiment, the choice was whether to solve the problems via mental arithmetic, calculator, or pencil and paper. Only elderly subjects participated in this experiment.

The reason for comparing these particular age groups on these particular strategy choices was curiosity about whether the sociohistorical changes introduced by new technology influence the strategy choice process. For current college students, calculators have been widely available throughout the time they were learning multiplication. In contrast, for current elderly adults, calculators did not exist when they initially learned multiplication, and they continued to be rare for many years thereafter. A great many old people continue either never to use them or only to use them occasionally, preferring to solve problems either with no external aid or with pencil and paper. Contrasting the choices made by undergraduates and by old people, and contrasting the choices of old people who have had more or less experience with calculators, promised to shed light on how the choice process is influenced by such sociohistorical changes.

In both the two-strategy experiment and the three-strategy experiment, subjects first solved a set of problems using whichever strategy they wanted (the choice condition); then, in the no-choice conditions, they solved comparable sets of problems needing either to use mental multiplication (no other tools present), or needing to use a calculator (they had to punch the buttons and get the answer even if they could retrieve the answer), or, in the three-strategy experiment, needing to use pencil and paper (again they needed to write out the problem and get the answer even if they could retrieve it). This design eliminated the biases in estimates of problem difficulty introduced by nonrandom assignment of which strategies were used and which subjects used them on each problem. It also allowed a direct test of whether the difference in the strengths of the strategies was the best predictor of how often each approach was used, as predicted by ASCM.

The college students, all of whom participated in the two-strategy experiment, were undergraduates at Carnegie Mellon. The elderly subjects in the two-strategy experiment averaged 66 years (range = 61–73 years). They were recruited from a senior citizen’s center. The elderly subjects in the three-strategy experiment averaged 75 years (range = 60–95), and were recruited from among the attendees of a different senior citizens’ center. Members of both elderly groups had occupied a wide range of occupations when they worked, including hairdresser, mail clerk, music teacher, bookkeeper, cashier, food-service worker, statistician, secretary, and maintenance worker. Both the undergraduates and the two elderly samples were
predominantly female (79% of the undergraduates; 87% and 97% of the two elderly samples).

In both procedures, subjects were presented four types of multiplication problems: N * 10 (e.g., 8 * 10), NN * 10 (e.g., 27 * 10), N * NN (e.g., 8 * 14), and NN * NN (e.g., 14 * 17). In the two-strategy experiment, a set of 72 problems was divided into three subsets of 24 problems each. The difficulty of problems was matched across the three sets, by equating mean product sizes as well as the types of problems within the sets. In the three-strategy experiment, comparable matching was done for a slightly different set of 72 problems divided into four subsets of 18 problems each. Presentation of the problem sets was counterbalanced in both experiments, so that each problem set appeared equally often in each condition.

**Variable Strategy Use**

As in previous studies, virtually all subjects, both younger and older, used multiple strategies. In the choice condition of the two-strategy experiment, 47 of 48 undergraduates and 59 of 60 elderly adults used both strategies. In the choice condition of the three-strategy experiment, 59 of 60 elderly adults also used at least 2 strategies, with 21 of 60 using all 3. Specifically, 1 subject only used mental arithmetic, 22 used mental arithmetic and the calculator, 16 used mental arithmetic and pencil and paper, and 21 used mental arithmetic, the calculator, and pencil and paper.

**Adaptive Strategy Choice**

Siegler and Lemaire (in press) examined three aspects of the adaptiveness of strategy choices, all of which pointed to similarly adaptive choices in younger and older adults.

**Having a Choice Enhanced Speed and Accuracy.** One test of the adaptiveness of choices involved examining whether being able to choose among alternative strategies yielded faster and more accurate performance than would have occurred if subjects had used the strategies in the same proportions but had assigned them randomly to problems. This test involved comparing speed and accuracy under no-choice and choice conditions. Each subject’s speed and accuracy in the no-choice conditions was weighted by that subject’s percent use of that strategy in the choice condition. Thus, for each subject’s latency, the projected RT in the no-choice condition of the two-strategy experiment was (Mean RT in calculator-required condition * Percent use of calculator in choice condition) + (Mean RT in mental-arithmetic-required condition * Percent use of mental arithmetic in choice condition). This comparison was performed both for all problems and separately for problems with and without 10 as a multiplicand.

As shown in Table 4.2, having a choice led to faster and more accurate performance than would have been projected solely from percent use of each strategy. Speed was greater for both problems that included 10 as a multiplicand and problems that did not. Accuracy was greater for problems that did not have 10 as a multiplicand; it was essentially perfect on problems with 10 as a multiplicand regardless of how it was computed. The pattern was identical for undergraduates and for elderly adults, and identical in both the two-strategy and the three-strategy experiment.

**Differences Between Performance Yielded by Strategies Predicts Their Frequency of Use.** ASCM predicts that the difference between the speeds and accuracies of alternative strategies should be a better predictor of their frequency of use than should any structural variable, such as the size of the product or the number of digits in the problem. To test this prediction, Siegler and Lemaire (in press) ran stepwise regression analyses of percent use of mental arithmetic on each problem. For the two-strategy experiment, the two predictor variables that would be expected to be especially predictive of percent use of each strategy were (a) difference between mean RT for each problem when it was presented in the mental-arithmetic-required
and calculator-required conditions; (b) difference between mean percent errors for each problem when it was presented in the mental-arithmetic-required and calculator-required conditions. The structural predictors that were included in the regression analyses were the sizes of the first multiplicand, second multiplicand, product, whether the problem included 10 as a multiplicand, and the number of digits in the problem.

In the analysis of college student's performance, three factors independently contributed to the percentage of variance accounted for in percent use of mental arithmetic on each problem: (a) difference between mean RT in the mental-arithmetic-required and calculator-required conditions on the problem ($R^2 = .72$); (b) whether 10 was a multiplicand (partial $R^2 = .14$); and (c) difference between mean percent errors in the mental-arithmetic-required and calculator-required condition on the problem (partial $R^2 = .01$). Overall, the three-term model explained 87% of the variance in frequency of use of mental arithmetic on each problem.

Similar results emerged in the two-strategy experiment with older adults. Three factors independently contributed to the percentage of variance accounted for in percent use of mental arithmetic on the problem: (a) difference between mean RT in the mental-arithmetic-required condition and the calculator-required condition on the problem ($R^2 = .61$); (b) whether one of the multiplicands was 10 (partial $R^2 = .18$); and (c) number of digits in the problem (partial $R^2 = .04$). Overall, the three-term model accounted for 83% of the variance. As with the undergraduates, most of the variance in the old people's frequency of use of mental arithmetic on the 72 problems was accounted for by the difference for each problem between performance yielded by the alternative strategies.

The analysis of the elderly people's percent use of mental arithmetic in the three-strategy experiment was slightly different, because here there were two alternatives to mental arithmetic. This led to Siegler and Lemaire including as a predictor not only the differences on each problem between speeds and accuracies generated by mental arithmetic and the calculator but also between those generated by mental arithmetic and pencil and paper. The same basic pattern of results emerged. Once again, the best predictor of percent use of mental arithmetic on each problem was a difference in performance between two strategies—in this case, the difference between percent errors on each problem generated using mental arithmetic and the calculator ($R^2 = .72$). Again problem characteristics added significant independent variance: number of digits (partial $R^2 = .07$) and whether 10 was a multiplicand (partial $R^2 = .06$). And once more, the three-term model accounted for a high percentage of total variance, in this case 85% of the variance in percent use of mental arithmetic on each of the 72 problems.

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Thus, in accord with ASCM's predictions, the differences in speed and accuracy generated by the strategies on each problem was an excellent predictor of the frequency with which the strategies were chosen on that problem. It predicted well in absolute terms, accounting for between 61% and 72% of the variance in the younger and older adults' frequency of choices of the mental arithmetic strategy, and also predicted more accurately than any of the structural features of the problems. By eliminating selection artifacts from estimates of the speed and accuracy generated by each strategy on each problem, the choice/no-choice design demonstrated just how closely strategy choices are attuned to the relative performance yielded by different strategies.

**Strategy Choices Were Almost Unbiased.** The choice/no-choice design also allowed independent estimates of bias in the strategy choices. This estimate was provided by the intercept term of the regression equations of predictors of strategy use. Even if percent use of mental arithmetic were perfectly correlated with the difference between the time required to solve the problem via mental arithmetic or a calculator, subjects might not split their choices evenly on problems on which the two strategies took equally long to execute. They might instead split them evenly when the times favored one strategy or the other by several seconds, thus indicating a bias toward that strategy. For example, they might have a constant bias of 5 seconds for the calculator, but alter their percent choice of the calculator by a constant 10% for every 1 second deviation from this bias. Thus, they would use the calculator on 40% of problems on which mental arithmetic was 4 seconds faster, 50% on which it was 5 seconds faster, and 60% on which it was 6 seconds faster. This analysis raised the possibility that even though the choices of elderly adults were just as systematic as those of younger ones, and predicted by almost identical variables, their choices might be more biased. In particular, given their limited experience with calculators, they might be more biased toward use of mental arithmetic.

In the two-strategy experiment, the intercept of both the undergraduates and the elderly adults differed significantly from 0, indicating that both groups of subjects were somewhat biased in their strategy choices. The bias in the two groups turned out to be in the same direction—in favor of mental arithmetic. It also proved to be almost identical in magnitude. The college students used each strategy on about 50% of trials when using the calculator yielded solutions about 1.5 sec faster than using mental arithmetic; the corresponding figure for older adults was 1.6 sec. Seen from a different perspective, on problems on which times were approximately equal (within 1 sec.) college students used mental arithmetic on 58% of trials, older adults on 59%. Thus, the biases of older and younger adults were no different.
Change

ASCM points to four main sources of strategic change: acquisition of new strategies, changing frequency of use of existing strategies, changing efficiency of execution of strategies, and changing adaptiveness of choices among strategies (Lemaire & Siegler, 1995). As described earlier, the present study did not examine discovery of new strategies, and both frequency of use of each strategy and degree of adaptiveness in the choices were highly similar for the younger and older adults. However, substantial changes with age were evident in efficiency of execution of the strategies.

To test changes in execution of each strategy, independent of differences in how often the strategies were used and which subjects used them, Siegler and Lemaire (in press) compared the speed and accuracy of younger and older adults in the no-choice condition of the two-strategy experiment.

Speed. Averaged across strategies and types of problems, younger adults were considerably faster than older adults (5.8 vs. 9.8 sec). They were faster both when they used mental arithmetic (7.3 vs. 11.1 sec) and when they used calculators (4.3 vs. 8.4 sec). The differences were greatest on the hardest problems (NN * NN) in the mental arithmetic condition. The undergraduates took an average of 13.5 sec to solve such problems, whereas the older adults required an average of 19.7 sec.

Accuracy of execution also was greater among the younger adults. The overall difference was 91% versus 86% correct. The difference was concentrated on the mental arithmetic problems (83% vs. 75% correct); both groups solved almost all problems correctly when they used the calculator. Somewhat surprisingly, differences in mental arithmetic accuracy were present on the NN * 10 problems (99% vs. 88% correct) as well as on the problems that did not include 10 as an addend. The fact that both speed and accuracy were greater among the younger adults indicated that the differences could not be attributed to speed-accuracy tradeoffs.

In summary, comparisons between younger and older adults revealed that (a) older adults were generally slower and less accurate than younger ones, but (b) the two groups used each strategy on comparable percentages of trials in the choice condition. This is consistent with several other studies of cognitive effects of aging, which have reported similar frequency of use of different strategies but less efficient execution of the strategies by older than by younger adults (e.g., Belmore, 1981; Cohen & Faulkner, 1983).

Individual Differences

In the three-strategy experiment, Siegler and Lemaire (in press) presented subjects with questionnaires concerning their past and present use of calculators and their attitudes toward the calculators, toward mental arith-

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metric, and toward solving problems via pencil and paper. Individual differences in these experiential and attitudinal variables were then related to individual differences in performance in the experimental situation.

As shown in Table 4.3, 33% of the elderly adults had no experience using calculators, 23% had occasional experience (less than once per month during both their working days and since), and the other 44% had more frequent experience. Degree of experience proved to be related to both how often and how efficiently subjects used the calculator. Percent use of the calculator increased from 19% for the elderly adults who never had used a calculator before the experiment to 30% among those who had used it more than once per month in either their work or current life. Even more striking were the changes with experience that occurred between choices of the calculator versus pencil and paper. Elderly adults who had never previously used a calculator used it on 40% of trials on which they used either a calculator or pencil and paper; those who had used it less than once per month chose it on 52% of trials on which they used one of the backup strategies, and those who had used it more than once per month chose it on 67% of trials on which they used one of the backup strategies.

CONCLUSIONS

As implied by ASCM, basic strategy choice phenomena seem to be present throughout the life span and in domains not usually thought of as cognitive, as well as ones that are. In particular, the experiments on infants’ locomotion and older adults’ multiplication indicated the same five phenomena as had previously been observed with children on standard cognitive tasks such as arithmetic, time telling, and memory strategies: cognitive variability, adaptive strategy choice, gradual change in strategy use, generalization of knowledge to new problems, and individual differences in style as well as expertise.

Cognitive variability was particularly evident in the infants’ strategies for going down risky hills. Sometimes they tried to walk down, sometimes they tried to crawl down, sometimes they slid down on their bellies face first, sometimes they slid down feet first, sometimes they slid down on their bottoms, sometimes they refused to go down at all. Similarly, almost all older adults, like almost all younger adults, used both mental arithmetic and calculators when those were the options. When pencil and paper was also an option, almost all older adults used at least two of the three approaches and 35% used all three.

Adaptive strategy choices were similarly apparent in both infants and older adults. Infants varied their GO ratios with their walking boundaries.
### Individual Differences in Older Adults' Arithmetic as a Function of Prior Experience With Calculators (Three-Strategy Experiments)

<table>
<thead>
<tr>
<th>Use of Calculators:</th>
<th>(N = 26)</th>
<th>(N = 14)</th>
<th>(N = 14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mental Arithmetic</td>
<td>52</td>
<td>44</td>
<td>55</td>
</tr>
<tr>
<td>Pencil and Paper</td>
<td>29</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Calculator</td>
<td>19</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Free Choice Condition</td>
<td>19</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>PA</td>
<td>10.7</td>
<td>9.0</td>
<td>9.4</td>
</tr>
<tr>
<td>NAR</td>
<td>94</td>
<td>72</td>
<td>75</td>
</tr>
<tr>
<td>Accuracy (Free Choice Condition)</td>
<td>29</td>
<td>26</td>
<td>28</td>
</tr>
</tbody>
</table>

**Table 4.3**

They almost always would try their usual mode of locomotion on hills they could successfully traverse, but greatly decreased their reliance on this approach as the hills became increasingly difficult for them to navigate in that way. Older adults showed similarly adaptive strategy choices. The more that considerations of speed and accuracy favored use of a given strategy, the more often they relied on that strategy in the choice condition. These adaptive choices also led to their speed and accuracy being greater in the choice conditions than in the no-choice ones.

Gradual changes in strategy use and strategy choice were also apparent in both infants and elderly people. With age and locomotor experience, infants progressively decreased their GO ratios on risky slopes, although not on safe slopes. This allowed them to decrease their frequency of falling on slopes where that was a danger, whereas maintaining the ease of their usual approach when the danger of falling was minimal. Similarly, older adults with more calculator experience used calculators on a greater percentage of backup strategy trials than did adults with little or no such experience. Even the older adults with the greatest calculator experience, however, still relied on pencil and paper on one-third of backup strategy trials, indicating that their changeover to that approach was sufficiently gradual as to be incomplete after many years of use.

The infants showed generalization of locomotor experience in their strategy choices for descending down risky slopes. Even though both older and younger infants had little if any experience going down risky slopes, the older infants nonetheless progressively decreased their GO ratios on the risky slopes while maintaining them near ceiling level on slopes that were at or below their walking boundary. Similarly, the older adults generalized from their experience of how to choose between using mental arithmetic or pencil and paper to choose between using mental arithmetic or a calculator.

Finally, individual differences were present along stylistic as well as competence dimensions. Infants differed not just in their walking boundaries—a measure of competence—but also in their degree of recklessness. Some infants would resort to backup strategies even on slopes that were only slightly beyond their walking boundaries; others would attempt to walk down even slopes that were well beyond their abilities. Similarly, older adults who had never previously used a calculator differed not only in their skill at mental arithmetic—a measure of competence—but also in their willingness to try calculators. Some of them never used the calculator when they had the option of using pencil and paper; others, equally unfamiliar with calculators, used calculators more often than the familiar pencil-and-paper approach. Thus, these five phenomena—variable strategy use, adaptive choices among strategies, gradual change, generalization of knowledge to unfamiliar problems, and individual differences in style as
well as competence—seem to be basic human characteristics present in infancy, childhood, young adulthood, and old age.

REFERENCES


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